Current Progress of Epicyclic Helical Channels for Parametric Ionization Cooling

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The milestones of EPIC study

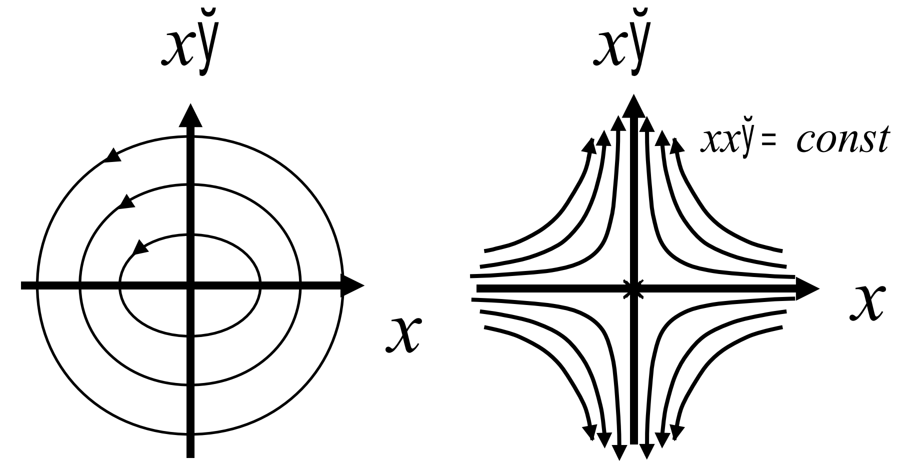
- **Theory:** To reach high luminosity without excessively large muon intensities, it was proposed to combine ionization cooling with techniques using parametric resonance (Y.Derbenev, R.Johnson, COOL2005 presentation; Advances in Parametric Resonance Ionization Cooling, ID: 3151 - WEPP149, EPAC08 Proceedings)
- **Mathematica simulations:** Preliminary simulation of EPIC optical properties for simplified magnetic fields (A.Afanasiev, Presentation: Epicyclic Helical Channels for PIC and REMEX, 13 Oct.2008)
- **Detailed G4beamline simulations:** Realistic EPIC design, emittance dynamics, absorbers, RF (V.Ivanov, at start point)

PIC Concept

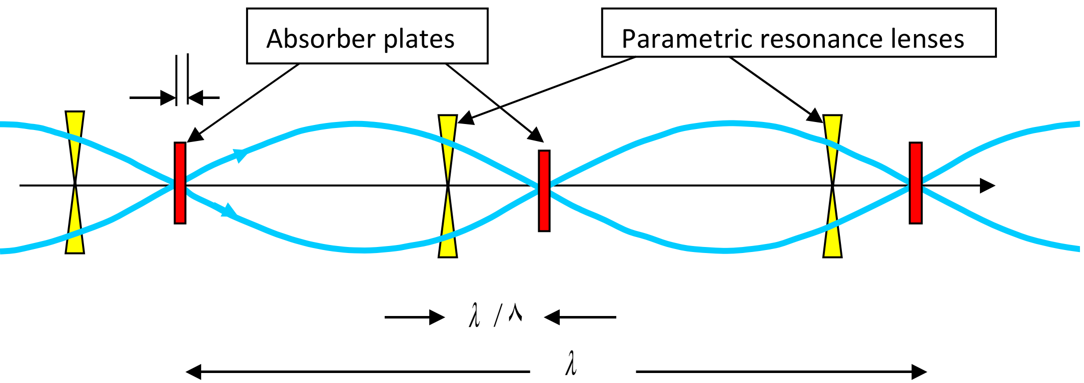
- Muon beam ionization cooling is a key element in designing next-generation high-luminosity muon colliders
- To reach high luminosity without excessively large muon intensities, it was proposed to combine ionization cooling with techniques using parametric resonance (Derbenev, JohnsonID: 3151 - WEPP149, these Proceedings)
- A half-integer resonance is induced such that normal elliptical motion of x-x’ phase space becomes hyperbolic, with particles moving to smaller x and larger x’ at the channel focal points
- Thin absorbers placed at the focal points of the channel then cool the angular divergence of the beam by the usual ionization cooling mechanism where each absorber is followed by RF cavities

PIC Concept (cont.)

Comparison of particle motion at periodic locations along the beam trajectory in transverse phase space



Ordinary oscillations vs Parametric resonance



Conceptual diagram of a beam cooling channel in which hyperbolic trajectories are generated in transverse phase space by perturbing the beam at the betatron frequency

PIC Challenges and solution

- Large beam sizes, angles, fringe field effects
- Need to compensate for chromatic and spherical aberrations
  - Requires the regions with *large* dispersion
- Absorbers for ionization cooling have to be located in the region of *small* dispersion to reduce straggling impact
- *Suggested solution (Derbenev, LEMC08; Afanasev, Derbenev, Johnson, EPAC08):*

  Design of an *epicyclic* HCC characterized by alternating dispersion and stability provided

  by solenoid with modified helix. Significant fringe fields are not present.

Helical Cooling Channel (HCC)

- Proposed to use for 6D muon cooling with homogeneous absorber, see for details

  Derbenev, Johnson, Phys.Rev.ST Accel.Beams 8, 041002 (2005)

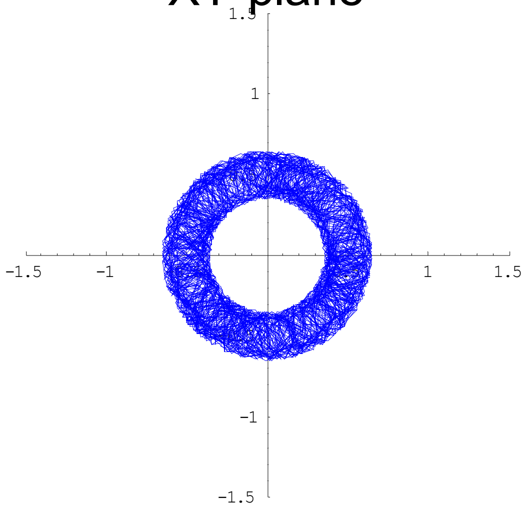
- Under development by Muons, Inc.
  - Y. Derbenev and R. Johnson, *Advances in Parametric Resonance Ionization Cooling*, ID: 3151 - WEPP149, these Proceedings
  - V. Kashikhin et al., *Design Studies of Magnet Systems for Muon Helical Cooling Channels,* ID: 3138 - WEPD015, these Proceedings

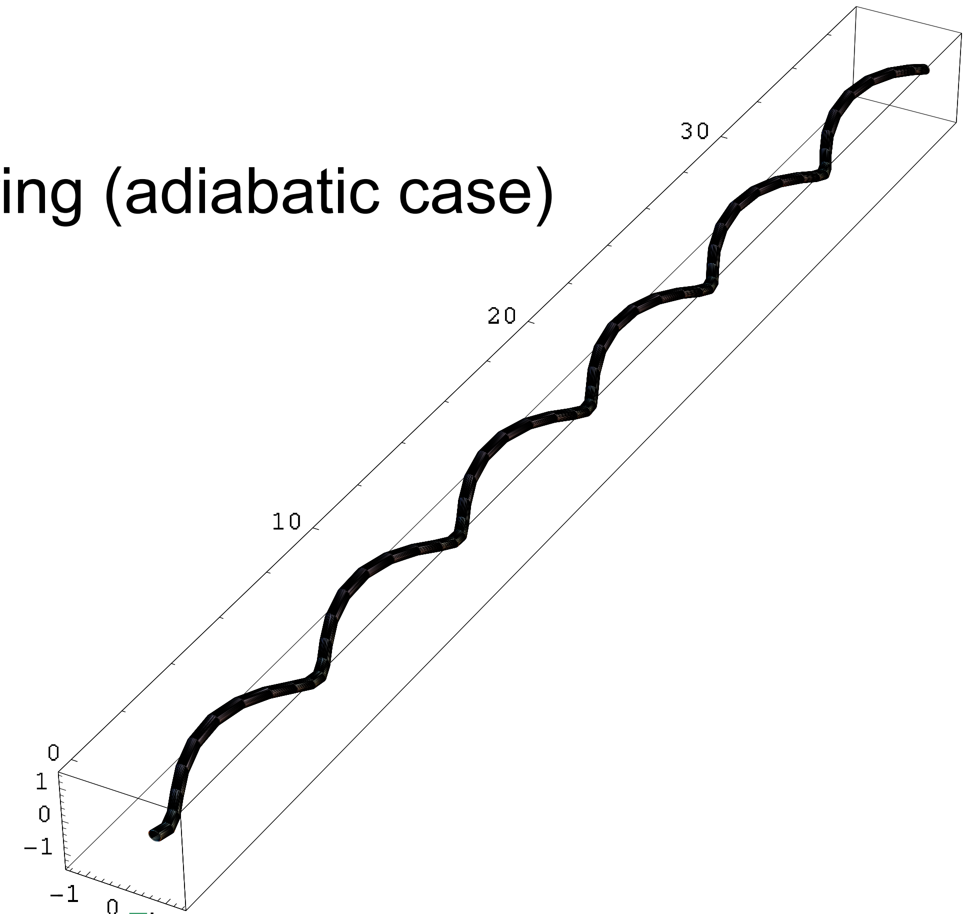
Helical Solenoid

$$B_z = B_s, \quad B_T = |B_{T\perp}| e^{ik_\perp z}$$

- *Derbenev, Johnson, Phys.Rev.ST Accel.Beams 8, 041002 (2005)*
- Dispersion= const
- Orbit is constrained by a ring (adiabatic case)

XY-plane

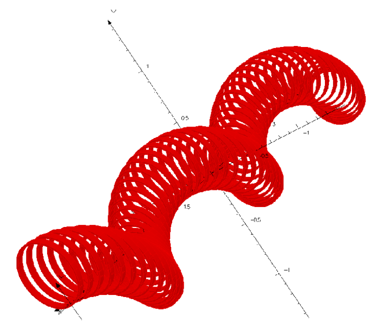




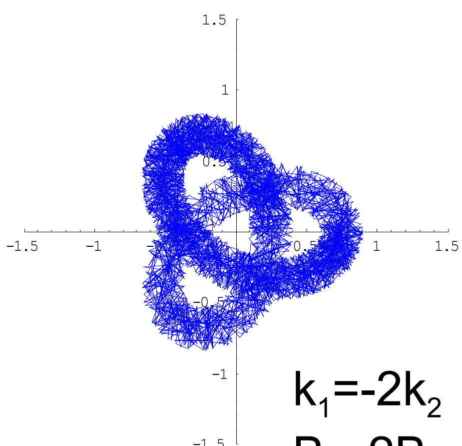
Epicyclic Helical Solenoid

$$B_T = |B_1| e^{ik_1 z} + |B_2| e^{ik_2 z}, B_z = const.$$

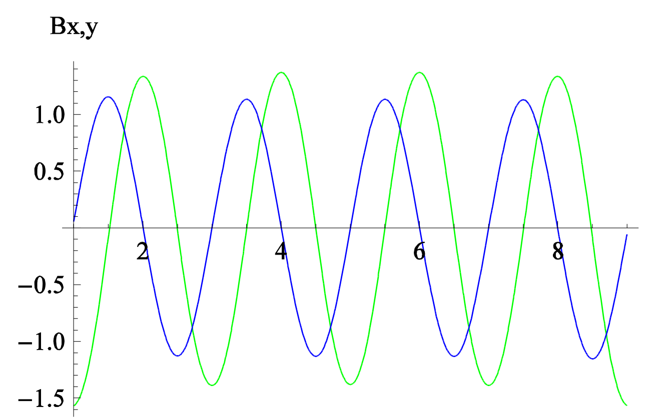
- **Superimposed transverse magnetic fields with two spatial periods**
- **Variable dispersion function**



XY-plane

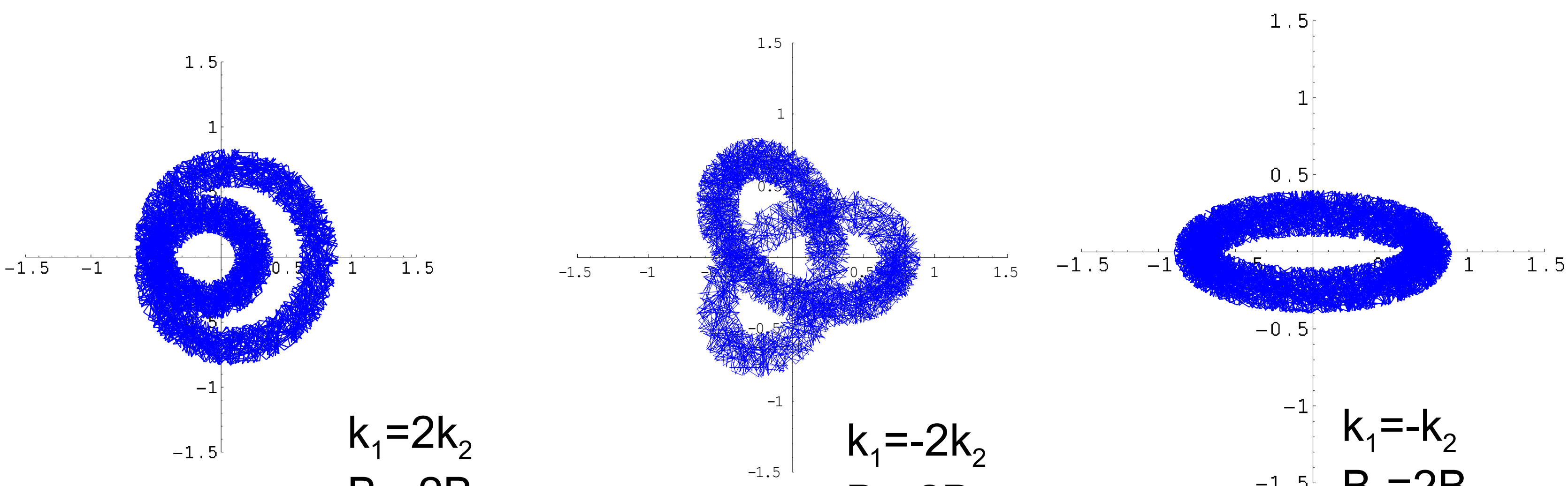


k_1=-2k_2
B_1=2B_2



Epicyclic Helical Solenoid (cont.)

- Wave numbers k_1,k_2 may be opposite-sign or same-sign
- Beam contained (in the adiabatic limit) by
  - Epitrochoid (same sign) - planet's trajectory in Ptolemy's system
  - Hypotrochoid (opposite-sign)- special case is an ellipse



$k_1=2k_2$
$B_1=2B_2$

$k_1=-2k_2$
$B_1=2B_2$

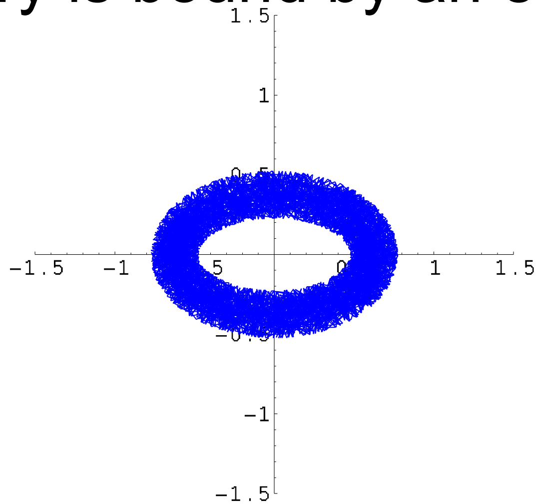
$k_1=-k_2$
$B_1=2B_2$

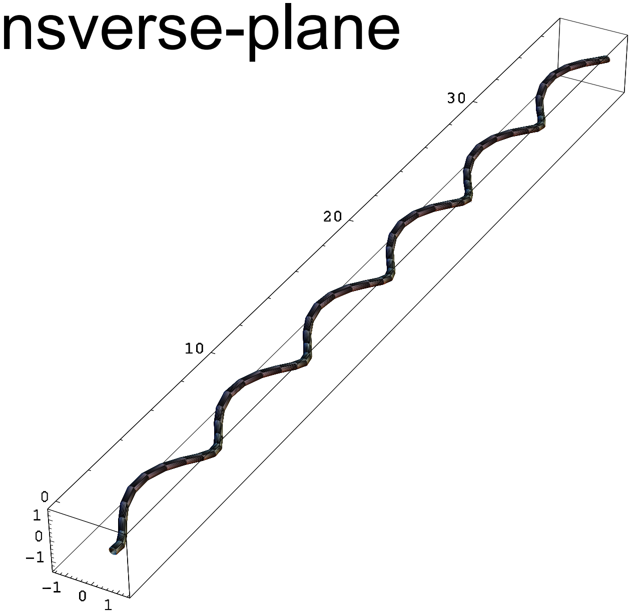
Oscillating Dispersion

- Derbenev's condition for nodes of dispersion function:

$$\frac{B_1}{(k_1 - k_c)^2} = \frac{B_2}{(k_2 - k_c)^2}, \quad k_1 \neq k_2$$

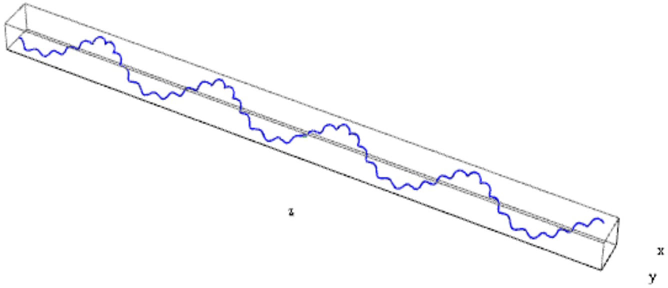
- For example, $k_1=-k_2=k_c/2$, then $B_2=9B_1$
- In the adiabatic limit $k_1=-k_2<<k_c$ transverse-plane trajectory is bound by an ellipse

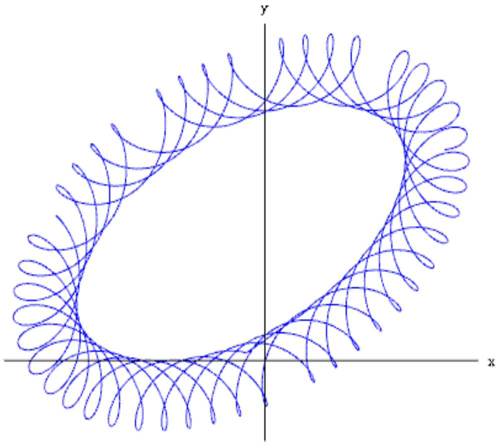




Particle Trajectory in Solenoid +Double-Periodic Transverse Field

- Particle 3D-trajectory in lab (upper plot) and its projection on the transverse plane (lower plot)
  - Adiabatic case: $k_1=-k_2<<k_c$.

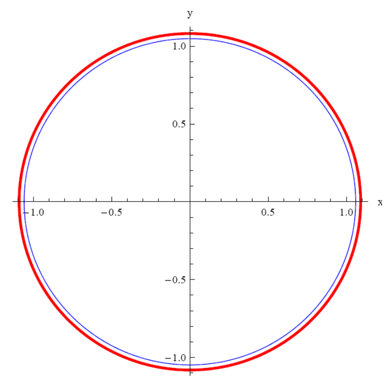




Orbit dispersion in Epicyclic HS

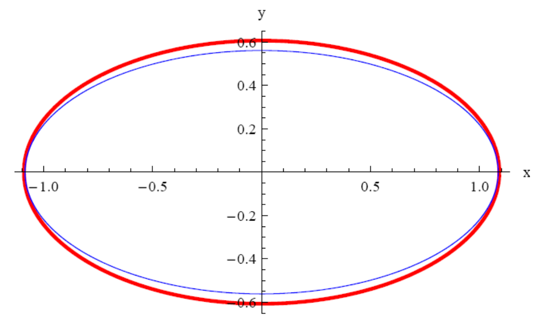
• Superimposed helical fields B_1+B_2 with two spatial periods:

$B_1 \neq 0$, $B_2=0$ (HS) → $B_1 \neq 0$, $B_2 \neq 0$ (Epicyclic HS)



p→p+Δp

$k_1=-k_2=k_c/2$



Variable dispersion function appears !

Change of momentum from nominal shows regions of zero dispersion and maximum dispersion

- Zero dispersion points: near plates for ionization cooling
- Maximum dispersion and beam size: Correction for aberrations

Implementing in PIC

- Plan to develop an epicyclic helical solenoid as part of PIC cooling scheme and Reverse-Emittance Exchange

- Elliptic option looks the simplest

- Detailed theory, numerical analysis and simulations are needed

The field model should be improved

Unfortunately simplified field distribution

$$B_x = B_1 \cos k_1 z + B_2 \cos k_2 z, \quad B_y = B_1 \sin k_1 z + B_2 \sin k_2 z, \quad B_z = B_0 = const;$$

doesn't satisfy the Laplace's equation. Let $B_2=\alpha B_1$, $k_1=-k_2$

Then $B_x=b(1+\alpha)\cos(kz)$, $B_y=b(1-\alpha)\sin(kz)$;

$B^2=b^2(1+\alpha)^2\cos^2 kz+b^2(1-\alpha)\sin^2 kz+B_0^2$,

$\Delta B^2=d^2/dz^2[b^2(1+\alpha)^2\cos^2 kz+b^2(1-\alpha)\sin^2 kz+B_0^2]= -2\alpha k^2 b^2[\cos^2 kz- \sin^2 kz]\neq 0.$

It means that field cannot be reproduced by the coils set exactly. We should add Bz modulation to get realistic field